MODULE 4.1

Competition

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The text's website has available for download for various system dynamics tools the file *sharkCompetition*, which contains a submodel for this module, available for download for various system dynamics tools.

Community Relations

In any population of organisms, the individuals are interacting with each other and with their environment. Populations, which are made up of only one species, are also interacting with other species in a particular area in what we term a **community**. These interactions influence the composition and dynamics of the community through time. Some of these interactions are robust, while others are not so robust or are even very weak. The magnitude of these interactions depends on the extent of their niche overlap. An **ecological niche** can be defined as the complete role that a species plays in an ecosystem. The more overlap two species have, the stronger the interaction will be. Two of these interactions between species are competition and predator-prey relationships.

Introduction to Competition

Everyone is familiar with competition. We compete for attention in families, for grades in school, for jobs and promotions, for parking spaces, and on and on. Competition is integral to most economic activity. Through competition in human societies, wages and prices are set; quantities and types of products manufactured are selected; businesses succeed or fail; and resources are distributed. Economic and social competition may occur even in noncapitalist systems.

More broadly, competition is a basic characteristic of all communities, human and nonhuman. It may occur within a population of the same species (**intraspecific**), like the human species, or it may occur between populations of different species (**interspecific**). Competitive interactions affect species distribution, community organization, and species evolution.

Simply speaking, **competition** is the struggle between individuals of a population or between species for the same limiting resource. If one individual (species) reduces the availability of the resource to the other, we term that type of competition **exploitative,** or **resource depletion**. This interaction is indirect and may involve removal of the resource or denial of living space. If there is direct interaction between individuals (species), where one interferes with or denies access to a resource, we term that competition **interference**. In this form, there may be physical contests for territory or resource. Interference may also, as in some plants, involve the production of toxic chemicals.

Modeling Competition

Sometimes two species are not eating each other but are competing for the same limited food source. For example, whitetip sharks (WTS) and blacktip sharks (BTS) in an area might feed on the same kinds of fish in a year when the fish supply is low. We anticipate that a large increase in one species, such as BTS, might have a detrimental effect on the ability of the other species, such as WTS, to obtain an adequate amount of food and, therefore, to thrive. Also, we expect that superior hunting skills of one species would diminish the food supply for the other species. As one species grows, the other shrinks, and vice versa.

In an unconstrained growth model (see Module 2.2, "Unconstrained Growth"), which ignores competition and limiting factors, we consider a population's (P) births to be proportional to the number of individuals in the population (r_1P) and its deaths to follow a similar proportionality (r_2P). Thus, in this model, the rate of change of the population is $dP/dt = r_1P - r_2P = (r_1 - r_2)P$, so that the solution is an exponential function, $P = P_0e^{(r_1 - r_2)t}$.

However, with competition, a competing species has a negative impact on the rate of change of a population. In this situation, we can model the number of deaths of each species as being proportional to its population size and the population size of the other species. Thus, for B being the population of BTS and W the population of WTS, the number of deaths of each species is proportional to the product BW. Moreover, the constant of proportionality associated with this proportionality for one species reflects competitive skills of the other species. (Projects explore various types of competition.) Consequently, we have the following equations for the change in the number of deaths of each species:

$$\Delta(\text{deaths of WTS}) = wBW, \text{ where } w \text{ is a WTS death proportionality constant}$$

$$\Delta(\text{deaths of BTS}) = bWB = bBW, \text{ where } b \text{ is a BTS death proportionality constant}$$

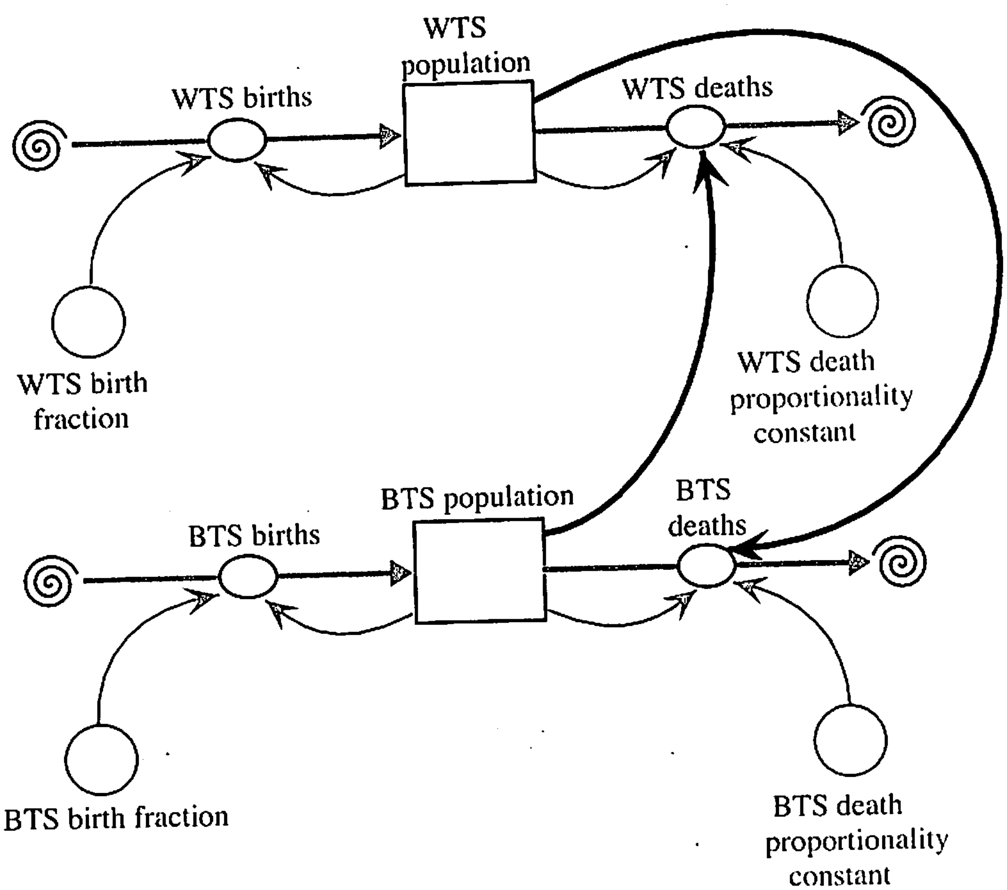

Figure 4.1.1 Model diagram of competition of species

Equation Set 4.1.1

Some equations to accompany Figure 4.1.1 with basic unit of time being 1 month

BTS_population(0) = 15
BTS_birth_fraction = 1
BTS_births = *BTS_birth_fraction* * *BTS_population*
BTS_death_proportionality_constant = 0.20
BTS_deaths = (*BTS_death_proportionality_constant* * *WTS_population*) * *BTS_population*
WTS_population(0) = 20
WTS_birth_fraction = 1
WTS_births = *WTS_population* * *WTS_birth_fraction*
WTS_death_proportionality_constant = 0.27
WTS_deaths = (*WTS_death_proportionality_constant* * *BTS_population*) * *WTS_population*

Figure 4.1.1 illustrates the interaction with the number of each species of shark affecting the deaths of the other species. With the basic unit of time being a month, Equation Set 4.1.1 gives some of the equations and constants, which in this case models births as being unconstrained. The set of numbers serve as an example and, although realistic, do not represent any actual population. Typically, a computational scientist uses actual field data to establish reasonable parameters for a model.

Quick Review Question 1

This question reflects on Step 2 of the modeling process—formulating a model—for developing a model for competition. As before, let W be the number of WTS and B the number of BTS. We simplify this model by assuming unconstrained births. After completing this question and before continuing in the text, we suggest that you develop a model for competition.

a. Give an equation for WTS births.
b. Give an equation for WTS deaths.

Quick Review Question 2

If all other parameters are equal and the WTS death proportionality constant (w) is larger than the BTS death proportionality constant (b), which population should be larger after a few time steps?

A. WTS B. BTS C. Impossible to determine

Note that in this hypothetical example, the death proportionality constants (0.2 and 0.27) are much smaller than the birth fractions (1 and 1). The former constants are multiplied by product of the two populations, BW, potentially a very large number, while the later are multiplied by their respective populations, B or W. For birth fractions of 1, each type of shark gives birth to approximately one pup each month. With *BTS_population*(0) being 15 and *WTS_population*(0) being 20, initial predictions are for about 15 BTS and 20 WTS to be born in the first month. Should a death proportionality constant for BTS or WTS also be 1, the rate of change of deaths for that type of shark would initially be $1 \times 15 \times 20 = 300$ sharks/month; and the population would quickly become extinct. Thus, we have the following rule of thumb.

Rule of Thumb: A constant of proportionality for a product of populations, such as BW, is frequently at least an order of magnitude (decimal point moved one place to the left) less than a constant of proportionality for one population, such as B or W.

With populations inhibited only by the competition for food, we might have a situation like the one illustrated in Figure 4.1.2 and Table 4.1.1. In this case, the WTS initially outnumber the BTS. However, the WTS death proportionality constant ($w = 0.27$) is larger than the BTS death proportionality constant ($b = 0.20$). Early in the simulation, the population of both species decreases. Eventually, the

WTS die out and the BTS thrive. The projects and exercises explore situations that have different initial populations and constants of proportionality and, consequently, different results.

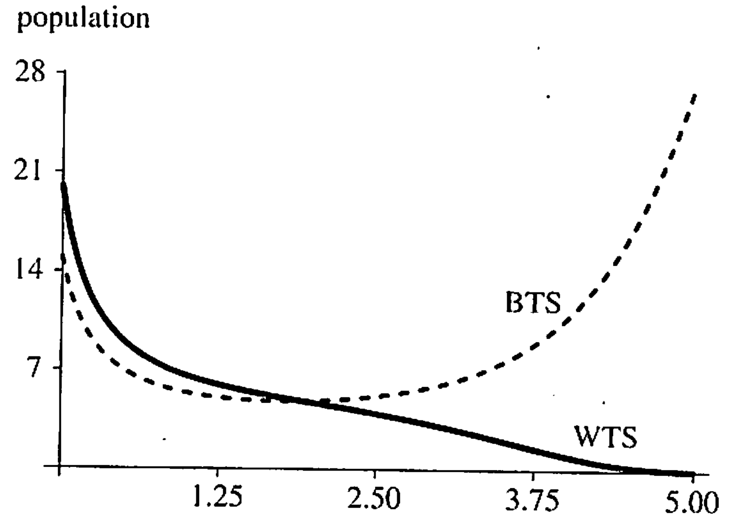

Figure 4.1.2 Graph of results of simulation from Figure 4.1.1, where the WTS death proportionality constant (w) is 0.27, the BTS death proportionality constant (b) is 0.20, and time (t) is in months

Table 4.1.1
Table of Results of Simulation from Figures 4.1.1 and 4.1.2 where $w = 0.27$ and $b = 0.20$

Time (months)	*WTS*	*BTS*
0	20.00	15.00
1	6.57	5.37
2	4.69	4.84
3	3.08	6.00
4	0.99	10.83
5	0.02	27.43

Exercises

1. a. Write the differential equations for modeling competition with unconstrained growth for both populations.
   b. Find all equilibrium solutions to these equations.
2. a. Write the differential equations for modeling competition with constrained growth for both populations.
   b. Find all equilibrium solutions to these equations.
3. What would be the effect on each of the following of increased intraspecific competition? *Hint:* Increased competition would be reflected in higher population densities.

a. Mortality in terms of number of pines/acre
b. Fertility in terms of number of seeds/plant/m^2
c. Average adult weight in terms of average adult bluegill weight per liter of water
d. Rate of growth in terms of increase in mallard duckling weight per unit of time

Projects

For additional projects, see Module 7.11, "Fueling Our Cells—Carbohydrate Metabolism."

1. a. Using your system dynamics tool's *sharkCompetition* file, which contains a model for competing species, find values for the initial populations and the constants of proportionality in which one population becomes extinct.
   b. Find values for which the two populations reach equilibrium.
   c. Discuss the results.
   d. Adjust the model to have the populations constrained by carrying capacities (see Module 2.3, "Constrained Growth").
   e. Adjust the parameters several times obtaining different results.
   f. Explain the models and discuss the results.
2. Argentine ants (*Linepithema humile*) are native to South America but have been invading the temperate zone of North America from the turn of the twentieth century. With its large and aggressive workers, Argentine ants are generally able competitively to exclude many native ant species. This success comes from the ant's ability to use exploitive as well as interference competitive mechanisms (Holway 1999).
   a. Develop a model of exploitive competition for the Argentine ant versus a native ant. The competitive factors include discovery time and rate of recruitment. The Argentine ant might discover a food source faster and attract other workers to the food source more quickly than the native ant.
   b. Develop a model of interference competition for the Argentine ant versus a native ant. The competitive factors include physical inhibition/removal and chemical repellents. Argentine ants might fight off or remove native ants from the food source, or they might use chemicals to repel them.
3. Model intraspecific competition. See Exercise 3 for examples. Discuss mortality and rate of growth in response to increasing intraspecific competition.
4. Plants can produce chemicals that, when released to the soil, inhibit the growth of other plants. These chemicals can act by inhibiting respiration, photosynthesis, cell division, protein synthesis, mineral uptake, or altering the function of membranes. For instance, sandhill rosemary (*Ceratiola ericoides*), an evergreen shrub found along the coastal plain of the southeastern United States, produces ceratiolin. This chemical washes from the leaves and degrades to hydrocinnamic acid, a compound that effectively inhibits seed germination of many competing species (Hunter and Menges 2002).

Assume that this chemical is increasingly effective at germination inhibition with increasing concentrations. Assume the highest concentration released to be 60 ppm (parts per million) and that concentration decreases linearly from the tips of the outermost leaves (for periods without rain).

a. Model inhibition of a competing plant species, where the effective concentrations of the toxin are between 20 and 60 ppm.

b. Model inhibition for this species with 2 cm rain per day. Set your own decrease in concentration per cm of rain for your model.

5. Model the interference competition of titmice versus other birds at feeders.
6. Model an environment with two competing species of flowering plants—species A and species B—and two essential resources—phosphorus and nitrogen. The constant renewal rate for each resource is 0.4 units/month. Initially, the availabilities of phosphorus and nitrogen are 12 units and 28 units, respectively. Each species has a starting population of 12 plants. At these levels, the maximum progeny produced per plant for species A and B are 1.2 plants/month and 1.0 plants/month, respectively; while their per plant deaths are 0.5 plants/month. Consider progeny production and deaths proportional to the number of species individuals. For maximum births, the phosphorus consumption amounts per plant for species A and B are 0.5/month and 0.25/month, respectively, and the nitrogen consumption amounts per plant are 0.25/month and 0.5/month, respectively. For fewer resources, the relative amounts of phosphorus and nitrogen consumption and the birth rates are proportionally smaller. Explain the model and discuss the results. Will this scenario result in equilibrium (Tilman 1980)?

Answers to Quick Review Questions

1. a. cW, where c is a birth rate

   b. wBW or wWB, where w is a death proportionality constant
2. A. BTS, because a larger portion of the white tip sharks are dying

References

Holway, David A. 1999. "Competitive Mechanisms Underlying the Displacement of Native Ants by the Invasive Argentine Ant." *Ecology*, 80(1): 238–251.

Hunter, Molly E., and Eric S. Menges. 2002. "Allelopathic Effects and Root Distribution of *Ceratiola ericoides* (Empetraceae) on Seven Rosemary Scrub Species." *American Journal of Botany*, 89(7): 1113–1118.

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MODULE 4.2

Predator-Prey Model

Download

The text's website has a *Predator-Prey* file, which contains the model of this module, available for download for various system dynamics tools.

Introduction

One of the interspecific interactions (see Module 4.1, "Competition") common to biological communities is the **predator-prey relationship**. When one species (**predator**) consumes another species (**prey**) while the latter is still living, the action is **predation**. Predation might involve the consumption of a young squirrel by a hawk, but examples also include tomato hornworms consuming tomato plant leaves and a tapeworm feeding off its mammalian host. Predator-prey interactions are important influences on population levels and ecosystem energy flow.

One of the most interesting characteristics of this type of relationship is that both predators and prey develop fascinating adaptations, which normally come about over long periods of time. Predator adaptations usually involve better prey detection and capture, whereas prey adaptations normally involve improved abilities to escape and avoid detection.

So, let's consider a 3/4-in. frog, commonly called a poison dart frog. We might expect that such a small animal would, to avoid predation, come out only at night or adopt some camouflaged coloration. However, this brazen creature forages for small invertebrates during the day (prey may also be predators) and is brilliantly colored (bright red, yellow, etc.). How might it manage then to avoid predation? The answer lies in the skin of the frog, which contains toxic, alkaloid chemicals that cause paralysis and/or death in the predator. Over time, predators associate the coloration with the toxic nature of the prey and, hence, avoid that prey. So the bright coloration is termed warning, or **aposematic, coloration**.

Lotka-Volterra Model

In the 1920s, mathematicians Vito Volterra and Alfred Lotka independently proposed a model for populations of a predator species and its prey, such as hawk and squirrel populations in a certain area. For simplicity, we assume that a hawk hunts only squirrels and that no other animal eats squirrels. If the hawk's only food source is squirrel and the number of squirrels diminishes significantly, then scarcity of food will result in starvation for some of the hawks. With reduced numbers of hawks, the squirrel population should increase.

Quick Review Question 1

This question reflects on the predator-prey situation before we begin the discussion.

- **a.** Do predator-prey interactions have a direct impact on the births or deaths of the prey?
- **b.** Based on other interaction model of Module 4.1, we can model the prey deaths as being directly proportional to what?
- **c.** If we consider prey births as being unconstrained, we can model prey births as being directly proportional to what?
- **d.** Are predator-prey interactions advantageous or disadvantageous for predators?
- **e.** Based on other interaction models of Module 4.1, we can model predator births as being directly proportional to what?
- **f.** If we consider predator deaths as being unconstrained, we can model the predator deaths as being directly proportional to what?

Let s be the number of squirrels in the area and h be the number of hawks. If no hawks are present, the change in s from time $t - \Delta t$ to time t is as in the unconstrained model (see Module 2.2, "Unconstrained Growth and Decay"):

$$\begin{aligned}\Delta s &= s(t) - s(t - \Delta t)\\ &= (\text{squirrel growth at time } t - \Delta t) * \Delta t\\ &= k_s * s(t - \Delta t) * \Delta t \text{ for constant } k_s\end{aligned}$$

However, this prey's population is reduced by an amount proportional to the product of the number of hawks and the number of squirrels, $h(t - \Delta t) * s(t - \Delta t)$. Thus, with a proportionality constant k_{hs} for this reduction, the change in the number of squirrels from time $t - \Delta t$ to time t is as follows:

$$\begin{aligned}\Delta s &= s(t) - s(t - \Delta t)\\ &= (\text{squirrel growth at time } t - \Delta t) * \Delta t\\ &= (k_s * s(t - \Delta t) - k_{hs} * h(t - \Delta t) * s(t - \Delta t)) * \Delta t\end{aligned}$$

for constants k_s and k_{hs}.

We can interpret the term $k_{hs} * h(t - \Delta t) * s(t - \Delta t)$ in a couple of ways. First, $h(t - \Delta t) * s(t - \Delta t)$ is the maximum number of distinct interactions of hawks with squirrels. For example, for $h(t - \Delta t) = 3$ hawks and $s(t - \Delta t) = 2$ squirrels, (3)(2) = 6 possible pairings exist. The decrease in the number of squirrels is proportional to this product, where the constant of proportionality, k_{hs}, is related to the hunting ability of the hawks and the survival ability of the squirrels. A second interpretation of $k_{hs} * h(t - \Delta t) * s(t - \Delta t) = (k_{hs} * h(t - \Delta t)) * s(t - \Delta t)$ is that the size of the squirrel population decreases in proportion to the size of the hawk population.

While the squirrel population decreases with more contacts between the predator and prey, the hawk population increases. Moreover, the death rate of hawks is proportional to the number of hawks. Thus, the change in the hawk population from time $t - \Delta t$ to time t is as follows:

$$\begin{aligned}\Delta h &= h(t) - h(t - \Delta t)\\ &= (\text{hawk growth at time } t - \Delta t) * \Delta t\\ &= (k_{sh} * s(t - \Delta t) * h(t - \Delta t) - k_h * h(t - \Delta t)) * \Delta t\end{aligned}$$

for constants k_{sh} and k_h. Although the deaths of the squirrels and the births of the hawks are both proportional to the product of the number of possible interactions of the two populations, their constants of proportionality, k_{hs} and k_{sh}, respectively, are probably different. For instance, 2% of the possible interactions might result in the death of a squirrel, while only 1% of the possible interactions might contribute to the birth of a hawk.

We can express the predator-prey model, known as the **Lotka-Volterra model**, as the following pair of difference equations for the change in prey (here, change in the squirrel population, Δs) and change in predator (here, change in the hawk population, Δh) from time $t - \Delta t$ to time t:

$$\begin{aligned}\Delta s &= (k_s * s(t - \Delta t) - k_{hs} * h(t - \Delta t) * s(t - \Delta t)) * \Delta t\\ \Delta h &= (k_{sh} * s(t - \Delta t) * h(t - \Delta t) - k_h * h(t - \Delta t)) * \Delta t\end{aligned} \tag{1}$$

or as the following pair of differential equations:

$$\begin{aligned}\frac{ds}{dt} &= k_s s - k_{hs} hs\\ \frac{dh}{dt} &= k_{sh} sh - k_h h\end{aligned} \tag{2}$$

Figure 4.2.1 contains a diagram for the predator-prey model with the prey population affecting the number of predator births and the predator population influencing the number of prey deaths.

Quick Review Question 2

Consider the following Lotka-Volterra difference equations:

$\Delta x = (2 * x(t - \Delta t) - 0.02 * y(t - \Delta t) * x(t - \Delta t)) * \Delta t$ with $x(0) = 100$

$\Delta y = (0.01 * x(t - \Delta t) * y(t - \Delta t) - 1.06 * y(t - \Delta t)) * \Delta t$ with $y(0) = 15$

a. Which equation (Δx, Δy, both, or neither) models the change in predator population?

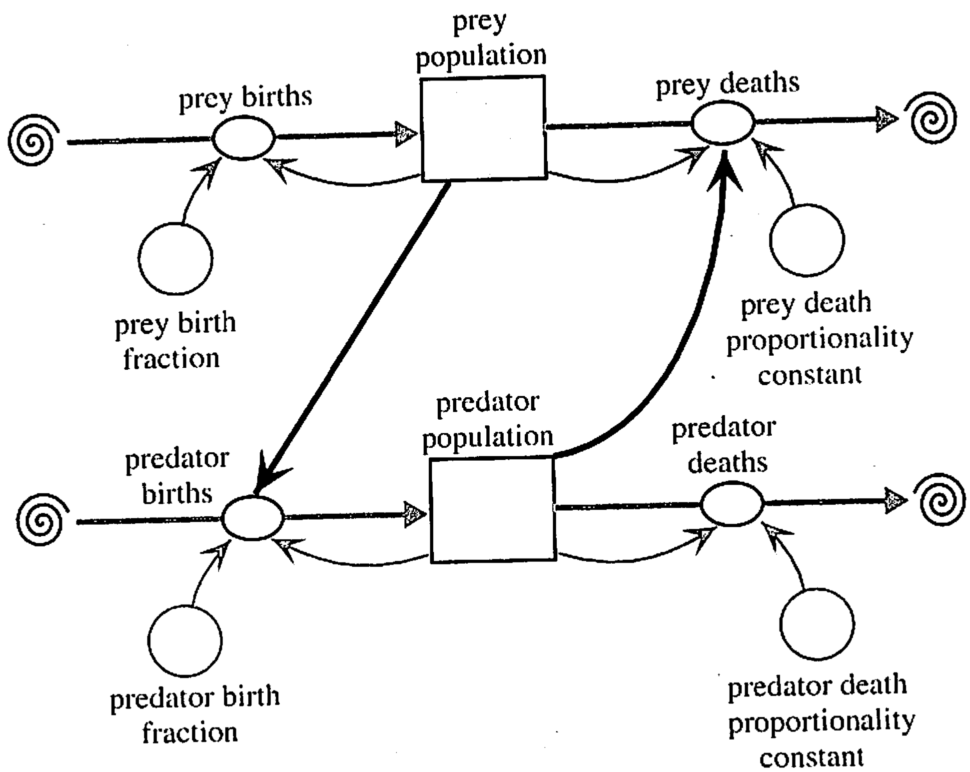

Figure 4.2.1 Predator-prey diagram

For each of the following questions, indicate the appropriate answer from the following choices:

A. 2	B. 0.02	C. −0.02	D. 0.01
E. 1.06	F. −1.06	G. 100	H. 15

b. Which number represents the predator birth fraction?
c. Which number represents the prey birth fraction?
d. Which number represents the predator death proportionality constant?
e. Which number represents the prey death proportionality constant?
f. What is the initial number of predators?
g. What is the initial number of prey?

Particular Situations

Historical Note During the Cultural Revolution in China (1958–1960), Chairman Mao Zedong decreed that all sparrows be killed because they ate too much of the crops and they seemed to be only for pleasure anyway. With reduction in its main predator, the insect population increased dramatically. The insects destroyed much more of the crops than the birds ever did. Consequently, the Chinese reversed the decision that caused the imbalance (PBS 2002).

Returning to the example of the hawks and squirrels, some of the model's equations and constants appear in Equation Set 4.2.1. In that example, *prey_birth_fraction* (k_s) = 2, *prey_death_proportionality_constant* (k_{hs}) = 0.01, *predator_birth_fraction* (k_{sh}) = 0.01, *predator_death_proportionality_constant* (k_h) = 1.06, the initial *prey_population* (s_0) = 100, and the initial *predator_population* (h_0) = 15. As suggested in the "Rule of Thumb" in Module 4.1, "Competition," the proportionality constants (0.01 and 0.01) for products, which involve interactions, are at least an order of magnitude less than the proportionality constants (2 and 1.06) for single populations.

Equation Set 4.2.1

Some of the equations and constants for model in Figure 4.2.1:

predator_population(0) = 15
predator_birth_fraction = 0.01
predator_births = (*predator_birth_fraction* * *prey_population*) * *predator_population*
predator_death_proportionality_constant = 1.06
predator_deaths = *predator_death_proportionality_constant* * *predator_population*
prey_population(0) = 100
prey_birth_fraction = 2
prey_births = *prey_birth_fraction* * *prey_population*
prey_death_proportionality_constant = 0.02
prey_deaths = (*prey_death_proportionality_constant* * *predator_population*) * *prey_population*

Table 4.2.1 and Figure 4.2.2 show the varying prey and predator populations as time advances through 12 months. Shortly after the squirrel, or prey, population

Table 4.2.1
Table of Prey and Predator Populations over 12-month period

Months	*Prey Population*	*Predator Population*
0.000	100.00	15.00
1.000	449.58	62.00
2.000	30.43	280.24
3.000	5.63	108.55
4.000	10.54	40.32
5.000	45.61	17.59
6.000	244.25	19.97
7.000	215.76	298.60
8.000	7.91	173.18
9.000	6.52	63.69
10.000	21.30	24.81
11.000	109.68	14.61
Final	470.44	74.28

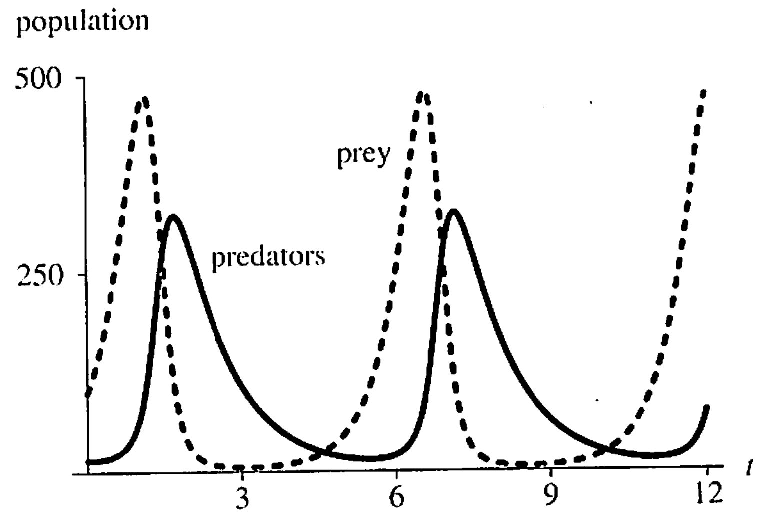

Figure 4.2.2 Graph of populations versus time in months

increases, the hawk, or predator, population does likewise. As the predators kill off their food supply, the number of predators decreases. Then, the cyclic process starts over.

Quick Review Question 3

To the nearest whole number, what is the period (in months) of the cyclic functions for population in Figure 4.2.2?

Figure 4.2.3 shows the graph of a solution to the difference or differential equations with the prey population along the horizontal axis and the predator population along the vertical axis. With the initial predator population being 15 and prey population being 100, the plot starts at the bottom toward the left and proceeds counterclockwise as time progresses. Initially, with few predators endangering them, the

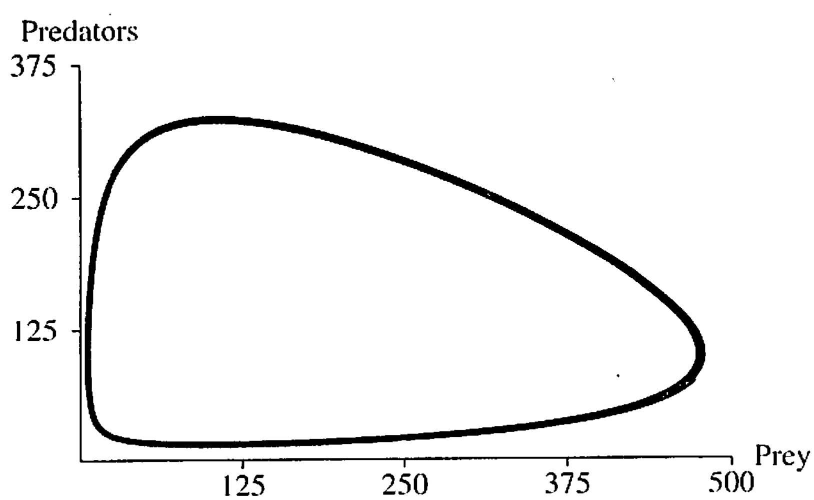

Figure 4.2.3 Graph of predator population versus prey population

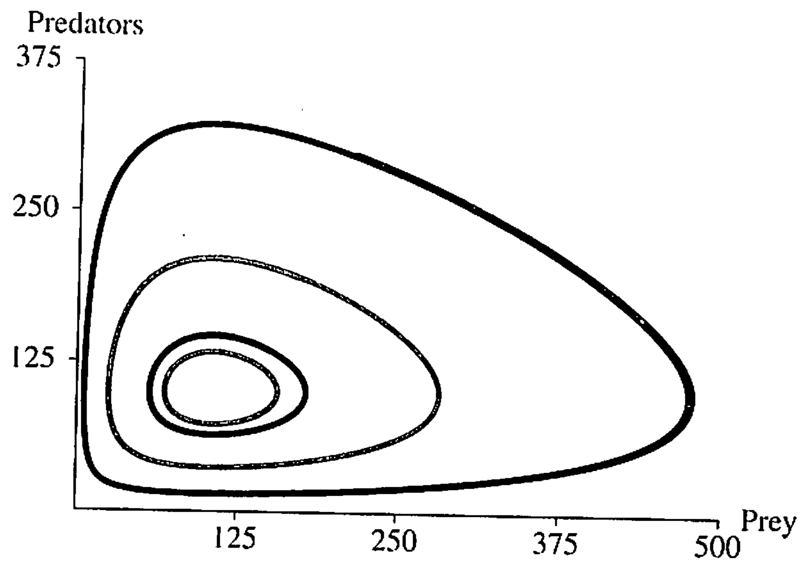

Figure 4.2.4 Several solutions to the predator-prey model using different initial conditions and the coloration shown

Predator	*Prey*	*Color of Graph*
15	100	black
75	125	gray
135	150	dark color
195	175	light color

prey population reaches a maximum of about 475 when the predator population is about 100. Then, with the graph developing to the left and up, we see that the prey population starts decreasing as the predator population continues to increase with the abundant supply of its food, the prey. At the graph's high point, about (107, 322), with approximately 107 prey, the predator population achieves a maximum of 322 individuals. That same number of predators, about 107, occurs toward the bottom of the graph when the prey only number about 15. After a maximum, the number of predators falls off rapidly because of the limited food supply, and the number of prey decreases as well. Eventually, on the bottom part of the graph, with the diminished number of predators, the prey are able to stage a comeback, and the cyclical process begins again. Figure 4.2.4 illustrates several such solutions employing different initial conditions.

Quick Review Question 4

The following are the Lotka-Volterra differential equations for the particular model we have been considering:

$$ds/dt = 2s - 0.02hs$$

$$dh/dt = 0.01sh - 1.06h$$

with $s(0) = 100$ and $h(0) = 15$.

a. Indicate all that must be true for the system to be in equilibrium: $ds/dt = 0$; $s = 0$; $dh/dt = 0$; $h = 0$; all of these; none of these.
b. A trivial solution for equilibrium is $s = 0$ and $h = 0$. Find a nontrivial solution, where $s \neq 0$ and $h \neq 0$.

Exercises

1. Give two sets of Lotka-Volterra equations with all coefficients being different that represent a system in equilibrium, such that the number of prey is always 3000 and the number of predators is always 500.
2. Write the differential or difference equations for a predator-prey model where there is a carrying capacity M for the predator. See differential equation 1 or difference equation 2 in Module 2.3, "Constrained Growth."
3. The blue whale, which can grow to 30 m in length, is a baleen whale whose favorite food is Antarctic krill, a small shrimp that is about 5 cm long. The difference equation for the change in the krill population is similar to that for Δs in (1), except the birth term must be logistic (see Equation 2 in Module 2.3, "Constrained Growth"). The difference equation for the change in the number of blue whales is a logistic equation, except that the carrying capacity is not a constant but is proportional to the krill population. Write the difference equations to model this system (Greenwood 1983)

Projects

For additional projects, see Module 7.11, "Fueling Our Cells—Carbohydrate Metabolism"; Module 7.12, "Mercury Pollution—Getting on Our Nerves"; Module 7.13, "Managing to Eat—What's the Catch?"; Module 7.14, "Control Issues: The Operon Model"; and Module 7.15, "Troubling Signals: Colon Cancer."

1. Develop a model where the prey birth fraction (k_s) is periodic, such as follows:

   $k_s = f + a\cos(p \times t)$, where f, a, and p are constants; $0 < a < f$; $0 < f$; and t is time.

   Note that a is the amplitude; the period is $2\pi/p$; and addition of f raises the graph of $a\cos(pt)$ by the amount f. (For a more detailed discussion, see the section "Trigonometric Functions" of Module 8.2, "Function Tutorial.") Have a table of population numbers, a graph of populations versus time, and a graph of one population versus the other. Determine values for the parameters so that the system is periodic, and then determine values where the system is chaotic. Discuss your results.
2. Using system dynamics software or a computer program, model the predator-prey example, including crop consumption discussed in the Historical Note about the Chinese Cultural Revolution.